Macroeconometrics

Developments, Tensions, and Prospects

Edited by
Kevin D. Hoover
University of California
Davis, California

Kluwer Academic Publishers
Boston / Dordrecht / London

Commentary on Chapter 11

Douglas G. Steigerwald

Frank Diebold and Jose Lopez have written an excellent primer on conditional heteroskedasticity (CH) models and their use in applied work. A principal motivation for CH models, as outlined in Diebold and Lopez, is their ability to parsimoniously capture the observed characteristics of many financial time series. By far the most widely used CH model, in part because of the fact that estimators for the model are simple to construct, is the generalized autoregressive conditional heteroskedasticity (GARCH) specification of order (1, 1) with normal innovations (henceforth termed the normal GARCH (1, 1) model). Despite its widespread use, the normal GARCH(1, 1) model does not account for important features in many financial time series. For example, assuming that the GARCH innovations have a normal density generates far fewer outliers than are typically observed in asset prices, while assuming that the order of the GARCH model is (1, 1) fails to account for the variety of dynamic patterns observed in the conditional heteroskedasticity of asset prices.

As Diebold and Lopez note in describing avenues for future research, it is important to consider alternative CH models that do account for such features of asset prices. Two alternatives to a normal GARCH(1, 1) model, which are mentioned by Diebold and Lopez and for which estimators are also simple to construct, are (1) to allow for nonnormal innovations that have a thicker tailed density, thereby accounting for a larger number of outliers and (2) to allow for orders other than (1, 1) by developing powerful test statistics for selection of order in GARCH models, thereby accounting for a wider variety of dynamic patterns. I discuss each of these alternatives in turn, in an effort to bring them within the set of commonly used methods for estimation and testing of CH models.

Unknown Density

Let y_t be a period-t variable (such as an exchange rate) that has conditional mean $x_t\beta$ where $x_t \varepsilon \Re^k$ and the period-t regressors include a constant. The normal GARCH(1, 1) model for y_t is

$$y_t = x_t\beta + h_t u_t, \tag{1}$$

where the period-t conditional variance is

$$h_t^2 = \omega + \alpha_1(y_{t-1} - x_{t-1}\beta)^2 + \gamma_1 h_{t-1}^2, \qquad (2)$$

with period-t innovation u_t and where $(\beta', \omega, \alpha_1, \gamma_1)$ are parameters to be estimated. The sequence $\{u_t\}_{t=1}^T$ is a sequence of independent and identically distributed (iid) normal random variables with mean zero and variance one.[1]

Because the normal GARCH(1, 1) model does not adequately account for outliers in asset prices such as exchange rates, researchers constructing CH models of exchange rates often assume that the density of u_t has thicker tails than a normal density. For example, Baillie and Bollerslev (1989) use both an exponential-power and a t density to model exchange rates.

Although the use of thicker-tailed parametric innovation densities does account for a larger number of outliers, it also raises the issue of the properties of the estimators if the selected density is misspecified. Virtually all researchers that estimate CH models also use a quasi-maximum likelihood estimator (QMLE). If the assumed density is normal, the QMLE is consistent for the parameters of the conditional variance. If the assumed density is nonnormal, then consistency of the QMLE depends on the specification of the conditional mean. For a nonnormal GARCH(1, 1) model, which is given by (1) and (2) together with the assumption that u_t has a nonnormal density, Newey and Steigerwald (1994) show that a nonnormal QMLE is not generally consistent.[2]

An alternative estimator that also accounts for a larger number of outliers is a semiparametric estimator. A semiparametric estimator of the parameters in a GARCH(1, 1) model is constructed under the assumption that the innovation density is any member within a class of densities, and uses a nonparametric estimator of the density. Steigerwald (1994) shows that a semiparametric estimator is consistent for general GARCH(p, q) models.

Given that a semiparametric estimator accounts for a larger number of outliers and consistently estimates the parameters of (1) and (2), attention turns to finite sample performance. The finite sample performance of a semiparametric estimator depends on the bandwidth used to construct the nonparametric density estimator. The bandwidth, in turn, depends on the conditional variance parameterization. For the conditional variance parameterization (2), the regularity conditions given in Steigerwald require that the bandwidth used to construct the nonparametric density estimator be smaller than the optimal bandwidth. Such a restriction on choice of bandwidth may lead to a poor estimate of the density, thereby reducing the gains of a semiparametric estimator.

A reparameterization of the conditional variance, which allows the optimal bandwidth to be used to estimate the density, is to let the variance of u_t be restricted only to be finite and to reparameterize the conditional variance as

$$h_t^2 = e^{\omega}[1 + \alpha_1(y_{t-1} - x_{t-1}\beta)^2] + \gamma_1 h_{t-1}^2. \qquad (3)$$

Linton (1993) develops this rep
Klaasen (1993) and Steigerwa
models.[3]

A guide to the finite sample p
two conditional varianc param
tained in Engle and Gonzalez-Ri
pare the performance of a semip
sample size of 2,000, Engle an
efficiency for a semiparametric e
density of u_t is a t density with 5
nonparametric estimator, report
estimator of the parameters in
a sample of only fifty observati
degrees of freedom. The efficien
indicating that the parameteriza
applied work.

Testing for Order

All discussion in the preceding
conditional variance. As is noted
fails to account for the variety
sponse, I turn to extending the G

To extend the GARCH(1, 1
statistic for choosing correct orc
tistics clear, I consider two disti
the null hypothesis that the condi
alternative hypothesis that the c
problem is to test the null hypo
against the multivariate alterna
ARCH($p + k$), where $k > 1$. In
testing the null hypothesis of h
ARCH(1) or ARCH(p).[4]

Engle (1982) develops Lagran
testing problem in a normal AR
thesis that a specific conditional
the alternative that the paramet
variance is always positive, the p
negative. Therefore, more powe
sided. For the univariate testing

Linton (1993) develops this reparameterization for ARCH models, Drost and Klaasen (1993) and Steigerwald extend the reparameterization to GARCH models.[3]

A guide to the finite sample performance of semiparametric estimators for the two conditional variance parameterizations is provided by the simulations contained in Engle and Gonzalez-Rivera (1991) and Steigerwald. Both studies compare the performance of a semiparametric estimator with a normal QMLE. For a sample size of 2,000, Engle and Gonzalez-Rivera report essentially no gain in efficiency for a semiparametric estimator of the parameters in (1) and (2) when the density of u_t is a t density with 5 degrees of freedom. Steigerwald, using a different nonparametric estimator, reports more favorable results for a semiparametric estimator of the parameters in (1) and (2), finding some efficiency gains with a sample of only fifty observations when the density for u_t is a t density with 5 degrees of freedom. The efficiency gains increase dramatically if (3) replaces (2), indicating that the parameterization of the conditional variance is important for applied work.

Testing for Order

All discussion in the preceding section considers a fixed order (1, 1) for the conditional variance. As is noted in the introduction, the (1, 1) order specification fails to account for the variety of dynamic patterns in many time series. In response, I turn to extending the GARCH(1, 1) specification to general order (p, q).

To extend the GARCH(1, 1) specification to GARCH(p, q) requires a test statistic for choosing correct order. To keep the following discussion of test statistics clear, I consider two distinct testing problems. The first problem is to test the null hypothesis that the conditional variance is ARCH(p) against the univariate alternative hypothesis that the conditional variance is ARCH$(p + 1)$. The second problem is to test the null hypothesis that the conditional variance is ARCH(p) against the multivariate alternative hypothesis that the conditional variance is ARCH$(p + k)$, where $k > 1$. In particular, the two problems can be viewed as testing the null hypothesis of homoskedasticity against the alternative either of ARCH(1) or ARCH(p).[4]

Engle (1982) develops Lagrange multiplier (LM) test statistics for the univariate testing problem in a normal ARCH model. The test is two sided, the null hypothesis that a specific conditional variance parameter equals zero is tested against the alternative that the parameter is nonzero. Yet to ensure that the conditional variance is always positive, the parameter of the conditional variance must be nonnegative. Therefore, more powerful test statistics can be constructed that are one sided. For the univariate testing problem, the signed square-root of the LM test statistic provides such a one-sided test. For the multivariate testing problem, there is no uniformly best one-sided test because the region over which the power function is evaluated spans more than one dimension. Lee and King (1993) propose a one-sided test statistic, termed an LBS test statistic, for the multivariate testing problem that maximizes the average slope, over all directions, of the power function in a neighborhood of the null hypothesis. They show that their one-sided test can be more powerful in finite samples than a two-sided LM test statistic.

Both the LM test statistic and the LBS test statistic are constructed under the assumption that the innovation density is normal. Fox (1994a) develops semiparametric versions of both test statistics. He finds that incorporating a nonparametric estimator of the density can have important finite sample consequences. Specifically, for samples of 100 observations the semiparametric test statistics achieve size-adjusted power gains of as much as 20 percent over their parametric counter-parts. Linton and Steigerwald (1994) extend the semiparametric tests to the reparameterization of the conditional variance in (3) and show that the semiparametric tests are optimal in that they maximize the average slope of the power function in a neighborhood of the null hypothesis for any innovation density in a general class. Fox also finds that testing for correct order is important in estimation, incorrect order specification can lead to substantial bias in the estimators of the conditional variance parameters.

Empirical Implementation

To demonstrate the potential importance of semiparametric methods in testing and estimation, I construct a model for the dollar per pound exchange rate. The data are collected at noon on the New York foreign exchange market and span the period January 2, 1985 to September 30, 1993 yielding 2,185 observations. As is commonly done, I model the first difference of the logarithm of the exchange rate rather than the exchange rate itself. The initial model is

$$y_t = \beta + h_t u_t,$$

where y_t is the period-t value of the change in the logarithm of the exchange rate and the conditional variance specification is given by

$$h_t^2 = e^{\omega}[1 + \alpha_1(y_{t-1} - \beta)^2] + \gamma_1 h_{t-1}^2.$$

Estimates of the parameters are reported in Table 11.1. (Standard errors are reported in parentheses below each estimate.) Although the magnitude of the normal QML and semiparametric estimates differ only slightly, the asymptotic standard errors of the semiparametric estimator are typically about half the size of the asymptotic standard errors for the normal QMLE. In addition, as Fox (1994b)

Table 11.1. Parameter estimates

Parameter	*Noı*
Beta	
Alpha	
Gamma	

notes, apparently small difference
ance parameters can have import
timal portfolio weights based on t
markedly for two sets of estimate
11.1. The portfolio weights impli
in the sense that the risk associate
return is reduced by 8 to 10 perc

Although the GARCH(1, 1) s
literature, it may not adequately
test for incorrect order specificati
variance is GARCH(1, 1) agains
variance is GARCH(2, 1). I const
of the LM test statistic and the Ki
semiparametric LM test statistics
hypothesis. The parametric King
the null hypothesis. Only the se
rejects the null hypothesis. It app
a GARCH(1, 1), the power gai
estimator and a one-sided altern

In summary, recent advance
researchers with powerful tools
normal GARCH(1, 1) model. S
available and provide alternative
of patterns in financial time seri

Notes

1. The variance of u_t is assumed t
separately identified.

Table 11.1. Parameter estimates for an exchange-rate model

Parameter	*Normal QML*	*Semiparametric*
Beta	−.0231	−.0227
	(.67[illegible]4)	(.3887)
Alpha	.06[illegible]3	.0702
	(.3540)	(.1533)
Gamma	.8884	.8876
	(.5638)	(.2742)

notes, apparently small differences in the point estimates of the conditional variance parameters can have important economic consequences. He shows that optimal portfolio weights based on the estimated conditional variance process differ markedly for two sets of estimates that differ only slightly, as do those in Table 11.1. The portfolio weights implied by the semiparametric estimators are better, in the sense that the risk associated with a portfolio that provides a fixed expected return is reduced by 8 to 10 percent out-of-sample.

Although the GARCH(1, 1) specification is common in the empirical finance literature, it may not adequately account for the dynamic pattern in the data. To test for incorrect order specification, I test the null hypothesis that the conditional variance is GARCH(1, 1) against the alternative hypothesis that the conditional variance is GARCH(2, 1). I construct both parametric and semiparametric versions of the LM test statistic and the King and Lee test statistic. Both the parametric and semiparametric LM test statistics, which are two-sided tests, fail to reject the null hypothesis. The parametric King and Lee test statistic also fails to clearly reject the null hypothesis. Only the semiparametric King and Lee test statistic clearly rejects the null hypothesis. It appears that if the true dynamic process is richer than a GARCH(1, 1), the power gains available from both a nonparametric density estimator and a one-sided alternative are needed to detect it.

In summary, recent advances in econometric methodology have provided researchers with powerful tools to move beyond the restrictive framework of a normal GARCH(1, 1) model. Semiparametric estimators and test statistics are available and provide alternatives that more flexibly account for the wide variety of patterns in financial time series.

Notes

1. The variance of u_t is assumed to equal one because $(\omega, \alpha_1, \gamma_1)$ and the scale of u_t are not separately identified.

2. To ensure that the likelihood has a unique maximum, which is a necessary condition for consistent estimation, the set of regressors must include the conditional standard deviation.

3. In (3) the parameter ω cannot be separately identified because the variance of u_t is restricted only to be finite, so only ratios of the parameters (namely $e^{\omega}\alpha_i$ and $e^{\omega}\gamma_1$) are identified.

4. Considering only ARCH processes is not restrictive, as Lee and King (1993) note testing a null hypothesis of homoscedasticity against an alternative hypothesis of ARCH(p) is equivalent to testing against an alternative of GARCH(p, q).

References

Baillie, R., and T. Bollerslev. (1989). "The Message in Daily Exchange Rates: A Conditional-Variance Tale." *Journal of Business and Economic Statistics* 7, 297–305.

Drost, F., and C. Klaasen. (1993). "Adaptivity in Semiparametric GARCH Models." Manuscript, Tilburg University.

Engle, R. (1982). "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of U.K. Inflation." *Econometrica* 50, 987–1008.

Engle, R., and G. Gonzalez-Rivera. (1991). "Semiparametric ARCH Models." *Journal of Business and Economic Statistics* 9, 345–360.

Fox, S. (1994a). "Semiparametric Testing of Generalized ARCH Processes." Manuscript, University of California, Santa Barbara.

Fox, S. (1994b). "Hedge Estimation Using Semiparametric Methods." Manuscript, University of California, Santa Barbara.

Linton, O., and D. Steigerwald. (1994). "Efficient Testing in GARCH Models." Manuscript, Yale University.

Lee, J., and M. King. (1993). "A Locally Mean Most Powerful Score Based Test for ARCH and GARCH Disturbances." *Journal of Business and Economic Statistics* 11, 17–27.

Linton, O. (1993). "Adaptive Estimation in ARCH Models." *Econometric Theory* 9, 539–569.

Newey, W., and D. Steigerwald. (1994). "Consistency of Quasi-Maximum Likelihood Estimators in Models with Conditional Heteroskedasticity." Manuscript, University of California, Santa Barbara.

Steigerwald, D. (1994). "Efficient Estimation of Financial Models with Conditional Heteroskedasticity." *Econometric Theory*, forthcoming.

12 DYI
AND TEST

Introduction

In the field of modeling econom
as the decade of cointegratio
econometricians alike invested
and empirical implications of N
time series of important econo
GNP may have statistical prope
the use of standard tools, suc
estimation.

The results of this research l
on almost every aspect of estima
series. It is therefore impossib
account of this field. The case
limited, given the several surv
Stock and Watson, 1988; Dola
Perron, 1991). The purpose of t
in my view, are important in a